

SIANO:

- i) f, g DERIV. IN $A =]a, x_0[$ o $A =]x_0, b[$; $a, b, x_0 \in \bar{\mathbb{R}}$
 ii) $\lim_{x \rightarrow x_0} f(x) = 0 = \lim_{x \rightarrow x_0} g(x)$ ($\Rightarrow f, g$ CONT. IN x_0)
 iii) $\forall x \in A, g'(x) \neq 0$ ($\stackrel{R}{\Rightarrow} \forall x \in A, g(x) \neq 0$)

ALLORA:

$$\lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)} = l \in \bar{\mathbb{R}} \quad \begin{matrix} \Rightarrow \\ \Leftarrow \end{matrix} \quad \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = l$$

DIM.

" \Leftarrow " $\lim_{x \rightarrow 0} \frac{x^2 \cos \frac{1}{x}}{x} = \lim_{x \rightarrow 0} x \cos \frac{1}{x} = 0$

$$\lim_{x \rightarrow 0} \frac{2x \cos \frac{1}{x} + \sin \frac{1}{x}}{1} \quad \text{N.E.}$$

" \Rightarrow " $x_0, l \in \mathbb{R}; \quad A =]a, x_0[$

$$\lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)} = l \Leftrightarrow \forall \varepsilon > 0, \exists a_\varepsilon \in]a, x_0[: \forall x \in]a_\varepsilon, x_0[, \left| \frac{f'(x)}{g'(x)} - l \right| < \varepsilon$$

$$\forall x \in]a_\varepsilon, x_0[, \exists c_x \in]x, x_0[: \frac{f(x)}{g(x)} = \frac{f(x) - f(x_0)}{g(x) - g(x_0)} = \frac{f'(c_x)}{g'(c_x)}$$

□

ES.

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\sin x}{6x} = \frac{1}{6}$$

□