

TEOREMA 5. - (DI ROLLE (1652-1719))

5.13

$$\left[ \begin{array}{l} \text{i) } f \text{ CONT. IN } [a, b] \\ \text{ii) } f \text{ DER. IN } ]a, b[ \\ \text{iii) } f(a) = f(b) \end{array} \right] \Rightarrow \exists c \in ]a, b[ : f'(c) = 0$$

DIM.

$$\text{i) } \xrightarrow{W.} \exists \bar{x}, \bar{\bar{x}} \in [a, b] : \forall x \in [a, b], f(\bar{x}) \leq f(x) \leq f(\bar{\bar{x}})$$

$$\begin{aligned} 1^{\circ} \text{ CASO: } \bar{x}, \bar{\bar{x}} \in \{a, b\} &\xrightarrow{iii} \forall x \in [a, b], f(\bar{x}) = f(a) = f(b) = f(\bar{\bar{x}}) = f(x) \\ &\Rightarrow \forall x \in ]a, b[, f'(x) = 0 \end{aligned}$$

$$2^{\circ} \text{ CASO: } \bar{x} \in ]a, b[ \xrightarrow{F.} f'(\bar{x}) = 0$$

$$3^{\circ} \text{ CASO: } \bar{\bar{x}} \in ]a, b[ \xrightarrow{F.} f'(\bar{\bar{x}}) = 0$$

□

LE i), ii), iii) SONO MINIMALI PER L'ESISTENZA DI UN PUNTO ESTREMALE:

$$\alpha) \alpha: x \in [0, 1] \rightarrow x \text{ VERIFICA i), ii) MA NON iii)}$$

$$\beta) \beta: x \in [-1, 1] \rightarrow |x| \text{ VERIFICA i), iii) MA NON ii)}$$

$$\gamma) \gamma: x \in [0, 1] \rightarrow \gamma(x) = \begin{cases} x & x \in [0, 1[ \\ 0 & x = 1 \end{cases}$$

VERIFICA ii), iii) MA NON i)

$$\text{E RISULTA: } \alpha'(x) \neq 0; \beta'(x) \neq 0; \gamma'(x) \neq 0 \quad \forall x$$

□