2° TEOREMA DI JEAN BERNOULLI - DE L'HOSPITAL

SIANO:

i) f, g DERIV. IN $A =]a, x_0[$ o $A =]x_0, b[$; $a, b, x_0 \in \overline{\mathbb{R}}$

ii) $\lim\limits_{x \to x_0} f(x) = \pm\infty = \lim\limits_{x \to x_0} g(x)$

iii) $\forall x \in A, \quad g'(x) \neq 0$

ALLORA:

$$* \quad \lim_{x \to x_0} \frac{f'(x)}{g'(x)} = l \in \overline{\mathbb{R}} \quad \underset{\not\Leftarrow}{\Longrightarrow} \quad \lim_{x \to x_0} \frac{f(x)}{g(x)} = l$$

DIM.

$$\text{"}\not\Leftarrow\text{"} \quad \lim_{x \to +\infty} \frac{x + \operatorname{sen} x}{x - \operatorname{sen} x} = \lim_{x \to +\infty} \frac{x\left(1 + \frac{\operatorname{sen} x}{x}\right)}{x\left(1 - \frac{\operatorname{sen} x}{x}\right)} = 1$$

$$\lim_{x \to +\infty} \frac{1 + \cos x}{1 - \cos x} \quad N.E.$$

$$\text{"}\Longrightarrow\text{"} \quad x_0, l \in \mathbb{R} \ ; \quad A =]a, x_0[$$

$$*; ii) \iff \forall \varepsilon > 0, \exists a_\varepsilon \in]a, x_0[: \forall x \in]a_\varepsilon, x_0[, \quad l - \frac{\varepsilon}{2} < \frac{f'(x)}{g'(x)} < l + \frac{\varepsilon}{2}$$

ED INOLTRE: $\quad f(x) \neq 0, \ g(x) \neq 0$

$$\text{CAUCHY} \Rightarrow \forall x \in]a_\varepsilon, x_0[, \exists c_x \in]a_\varepsilon, x[: \frac{f(x) - f(a_\varepsilon)}{g(x) - g(a_\varepsilon)} = \frac{f'(c_x)}{g'(c_x)}$$

$$\Rightarrow \forall x \in]a_\varepsilon, x_0[, \quad l - \frac{\varepsilon}{2} < \frac{f(x) - f(a_\varepsilon)}{g(x) - g(a_\varepsilon)} = \frac{f(x)\left(1 - \frac{f(a_\varepsilon)}{f(x)}\right)}{g(x)\left(1 - \frac{g(a_\varepsilon)}{g(x)}\right)} < l + \frac{\varepsilon}{2}$$

BUONCORE

ORA SI HA:

$$\lim_{x \to x_0} \frac{1 - \frac{g(a_\varepsilon)}{g(x)}}{1 - \frac{g(a_\varepsilon)}{g(x)}} = 1 > 0 \ ; \quad l - \varepsilon < \left(l - \frac{\varepsilon}{2}\right) \frac{1 - \cdots}{1 - \cdots} \ ; \left(l + \frac{\varepsilon}{2}\right) \frac{\cdots}{\cdots} <$$

E QUINDI:

$$(l - \varepsilon) < \left(l - \frac{\varepsilon}{2}\right) \frac{1 - \frac{g(a_\varepsilon)}{g(x)}}{1 - \frac{f(a_\varepsilon)}{f(x)}} < \frac{f(x)}{g(x)} < \left(l + \frac{\varepsilon}{2}\right) \frac{\cdots}{\cdots} < l + \varepsilon \qquad \square$$