TEOREMA 9. -

$$\left[\begin{array}{l} i)\ I = \text{INTERVALLO} \\ ii)\ f \text{ CONT. IN } I \\ iii)\ f'(x) = 0,\ \forall x \in \mathring{I} \end{array}\right] \Longrightarrow f(x) = \text{cost.},\ \forall x \in I$$

DIM.

$x_0 \in I$; $\quad \forall x \in I - \{x_0\},\ \exists c \in]\min\{x, x_0\}, \max\{x, x_0\}[$:

$$f(x) - f(x_0) = f'(c)(x - x_0) = 0$$

$$\Rightarrow \forall x \in I - \{x_0\},\quad f(x) = f(x_0)$$

$\square$

ES.

$$f: x \in A = [0,1] \cup [2,3] \longrightarrow f(x) = \begin{cases} 1 & \text{SE } x \in [0,1] \\ 2 & \text{SE } x \in [2,3] \end{cases}$$

VALGONO $ii)$ E $iii)$, NON $i)$.

$\square$

$$\left[F: I \to \mathbb{R} \text{ PRIMITIVA DI } f: I \to \mathbb{R} \right] \overset{\text{DEF}}{\Longleftrightarrow} \left[\forall x \in I,\ F'(x) = f(x) \right]$$

TEOREMA 10.- (CARATTERIZZAZIONE DELLE PRIMITIVE)

SE F È UNA PRIMITIVA DI f IN I, ALLORA

$$\left[G \text{ PRIMITIVA DI } f \text{ IN } I \right] \Longleftrightarrow \left[\exists k = \text{cost.} : G = F + k \right]$$

BUONOCORE

DIM.

"$\Rightarrow$" $\left[F' = f ;\ G' = f \right] \Rightarrow (G - F)'(x) = G'(x) - F'(x) = 0,\ \forall x \in I$

$$\Rightarrow G - F = k$$

"$\Leftarrow$" $G' = F' + 0 = f$

$\square$

$$\int f(x)\,dx = F(x) + c$$