

TEOREMA 6. - (DI DARBOUX (1842-1917))

$$f \text{ CONTINUA IN } I \text{ INTERVALLO} \Rightarrow \left[ \begin{array}{l} \forall [a, b] \subseteq I, \\ \forall \lambda \in ] \inf_{[a, b]} f(x), \sup_{[a, b]} f(x) [ \\ \exists c \in [a, b] : f(c) = \lambda \end{array} \right]$$

DIM.

$$\inf_{[a, b]} f(x) < \lambda < \sup_{[a, b]} f(x) \Rightarrow \exists a_1, b_1 \in [a, b] :$$

$$: \inf_{[a, b]} f(x) \leq f(a_1) < \lambda < f(b_1) \leq \sup_{[a, b]} f(x)$$

$$\Rightarrow \varphi(a_1) = f(a_1) - \lambda < 0 < f(b_1) - \lambda = \varphi(b_1)$$

$$\Rightarrow \exists c \in ]a_1, b_1[ : 0 = \varphi(c) = f(c) - \lambda$$

$$\Rightarrow f(c) = \lambda$$

□

TEOREMA 7. - (DI BOLZANO (1781-1848))

$$f \text{ CONTINUA IN } I \text{ INTERVALLO} \Rightarrow f(I) \text{ INTERVALLO}$$

DIM.

DAL TEOREMA 6 CON  $[a, b] = I$

□