TEOREMA 5.- (DEGLI ZERI)

$$\left[\begin{array}{l} i)\ f \in C^0[a,b] \\ ii)\ \exists\, a_1, b_1 \in [a,b] :\ f(a_1)\cdot f(b_1) < 0 \end{array}\right] \Rightarrow \exists\, c \in \,]a_1, b_1[\, :\ f(c) = 0$$

DIM.

(METODO DI INSCATOLAMENTO)

$f(a_1) < 0 < f(b_1)$

$$I_2 = [a_2, b_2] = \begin{cases} [a_1, c_1] & \text{SE} \quad f(a_1) < 0 < f(c_1) \\ [c_1, b_1] & \text{SE} \quad f(c_1) < 0 < f(b_1) \end{cases}$$

$\vdots$

$$I_n = [a_n, b_n] : \quad \begin{array}{l} j)\ \ I_{n+1} \subset I_n \ \text{ CIOÈ } \ a_n \le a_{n+1} \le b_n \le b_{n+1} \\ jj)\ \ b_n - a_n = \dfrac{b_1 - a_1}{2^{n-1}} \\ jjj)\ \ f(a_n) < 0 < f(b_n) \end{array}$$

$$\left.\begin{array}{r} jjj)\ \ f(a_n) < 0 < f(b_n) \\ j), jj) \Rightarrow \lim a_n = \sup a_n = c = \inf b_n = \lim b_n \end{array}\right] \Rightarrow$$

$$\Rightarrow \quad \lim f(a_n) \le 0 \le \lim f(b_n)$$

$$\begin{array}{ccc} \lim f(a_n) & & \lim f(b_n) \\ \| & & \| \\ \lim\limits_{x \to c} f(x) & & \lim\limits_{x \to c} f(x) \\ \| & & \| \\ f(c) & & f(c) \end{array}$$

$\square$

BUONOCORE