TEOREMA "PONTE". -

$$\lim_{x \to x_0} f(x) = \ell \in \overline{\mathbb{R}} \iff \alpha) \left[\begin{array}{c} \forall \{x_n\}_{n \in \mathbb{N}} : x_n \in A - \{x_0\} \text{ e } \\ \lim x_n = x_0 \text{ RISULTI: } \\ \lim f(x_n) = \ell \end{array} \right]$$

DIM.

"$\Longrightarrow$"

$$\left. \begin{array}{l} i) \lim_{x \to x_0} f(x) = \ell \iff \forall J(\ell), \exists I(x_0) : f(A \cap I - \{x_0\}) \subset J \\ ii) \lim x_n = x_0 \iff \forall I(x_0), \exists \nu \in \mathbb{N} : \forall n > \nu, \quad x_n \in I \\ iii) \forall n \in \mathbb{N}, \quad x_n \neq x_0 \end{array} \right] \Longrightarrow$$

$$\Longrightarrow \forall J(\ell), \exists \nu \in \mathbb{N} : \forall n > \nu, \quad f(x_n) \in J \iff \lim f(x_n) = \ell$$

"$\Longleftarrow$"

p.a. $\exists J(\ell) : \forall I(x_0), \exists \bar{x} \in A \cap I - \{x_0\} : f(\bar{x}) \notin J \Rightarrow$

$$\Rightarrow \exists J(\ell) : \forall n \in \mathbb{N}, \exists x_n \in A \cap]x_0 - \tfrac{1}{n}, x_0 + \tfrac{1}{n}[- \{x_0\} : f(x_n) \notin J$$

$$\Rightarrow \begin{cases} \{x_n\}_{n \in \mathbb{N}} : x_n \in A - \{x_0\} \\ \lim x_n = x_0 \\ \lim f(x_n) \neq \ell \qquad \text{ASSURDO!} \end{cases}$$

$\square$