TEOREMA 7.- (OPERAZIONI SUI LIMITI)

$$\left[\begin{array}{l} \lim a_n = a \in \mathbb{R} \\ \lim b_n = b \in \mathbb{R} \end{array}\right] \Longrightarrow \left[\begin{array}{l} \text{j)} \ \lim (a_n \pm b_n) = a \pm b \\ \text{jj)} \ \lim (a_n b_n) = a b \\ \text{jjj)} \ \lim \dfrac{1}{b_n} = \dfrac{1}{b} \quad \text{SE } b \neq 0 \end{array}\right]$$

DIM.

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$$|(a_n \pm b_n) - (a \pm b)| = |(a_n - a) \pm (b_n - b)| \leq$$

$$\leq |a_n - a| + |b_n - b| < 2\varepsilon$$

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$$\lim b_n = b \in \mathbb{R} \overset{\text{th. 2}}{\Longrightarrow} \exists k > 0 : \ \forall n \in \mathbb{N}, \ |b_n| \leq k$$

$$\Longrightarrow |a_n b_n - a b| = |a_n b_n - a b_n + a b_n - a b| \leq$$

$$\leq |b_n| \cdot |a_n - a| + |a| \cdot |b_n - b| <$$

$$< k\varepsilon + |a| \varepsilon = (k + |a|)\varepsilon$$