TEOREMA 2. -

$$\lim a_n = l \in \mathbb{R} \underset{\not\Leftarrow}{\Longrightarrow} \{a_n\}_{n\in\mathbb{N}} \text{ LIMITATA}$$

DIM.

"$\not\Leftarrow$" $\{(-1)^n\}_{n\in\mathbb{N}}$; $-1 \le (-1)^n \le 1$, $\forall n\in\mathbb{N}$

"$\Rightarrow$"

$$\left.\begin{array}{l} i)\ \lim a_n = l \iff \forall \varepsilon>0, \exists \nu\in\mathbb{N} : \forall n>\nu,\ l-\varepsilon < a_n < l+\varepsilon \\ ii)\ \text{PONIAMO: } h = \min\{a_1,\dots,a_\nu, l-\varepsilon\} \\ \qquad\qquad\quad k = \max\{a_1,\dots,a_\nu, l+\varepsilon\} \end{array}\right] \Longrightarrow$$

$$\Longrightarrow \forall n\in\mathbb{N},\quad h \le a_n \le k$$

□

TEOREMA 3. -

$$\lim a_n = +\infty\ (-\infty) \underset{\not\Leftarrow}{\Longrightarrow} \left[\begin{array}{l}\{a_n\} \text{ LIM. INF. (LIM. SUP.)} \\ \quad \text{E NON LIM. SUP. (NON LIM. INF.)}\end{array}\right]$$

DIM. "$\Rightarrow$"

$$\left.\begin{array}{l} i)\ \lim a_n = +\infty \iff \forall \varepsilon>0, \exists \nu\in\mathbb{N} : \forall n>\nu,\ \varepsilon < a_n \\ ii)\ \text{PONIAMO } h = \min\{a_1,\dots,a_\nu, \varepsilon\} \end{array}\right] \Rightarrow \left[\begin{array}{c}\forall n\in\mathbb{N} \\ h\le a_n\end{array}\right]$$

"$\not\Leftarrow$" $a_n = \begin{cases} n & \text{se } n \text{ PARI} \\ 1/n & \text{se } n \text{ DISPARI} \end{cases}$

□