(1781-1848) - (1813-1897)

Teorema di Bolzano-Weierstrass :

A infinito e limitato $\Longrightarrow$ $D(A) \neq \phi$

Dim. (Metodo di inscatolamento)

A limitato $\Rightarrow \exists\, a_1, b_1 \in \mathbb{R}$: $\quad A \subseteq [a_1, b_1]$

$$\Big(A \cap [a_1, e_1]\Big) \cup \Big(A \cap [e_1, b_1]\Big) = A \quad \text{infinito}$$

$[a_2, b_2] \cap A$ infinito ; $\quad a_1 \leq a_2 < b_2 < b_1$; $\quad b_2 - a_2 = \dfrac{b_1 - a_1}{2}$

$\vdots$

$[a_n, b_n] \cap A$ infinito ; $\quad a_1 \leq \ldots \leq a_n \leq a_{n+1} < b_{n+1} \leq b_n \leq \cdots \leq b_1$

$$b_{n+1} - a_{n+1} = \frac{b_1 - a_1}{2^n} \Rightarrow \lim (b_n - a_n) = 0$$

$\Longrightarrow \lim a_n = \sup a_n = x_0 = \inf b_n = \lim b_n$

$\overset{)=0}{\Longrightarrow} \forall \delta > 0, \ \exists\, [a_n, b_n] \subset \,]x_0 - \delta, x_0 + \delta[$

$\Rightarrow \forall \delta > 0, \ \ ]x_0 - \delta, x_0 + \delta[\, \cap A \supseteq [a_n, b_n] \cap A$ infinito

$\Rightarrow \ x_0 \in D(A)$.

$\square$