TEOREMA 4.- (REGOLARITÀ DELLE SUCCESSIONI MONOTONE)

$$\{a_n\}_{n\in\mathbb{N}} \text{ MONOTONA} \Rightarrow \exists \lim a_n$$

E PIÙ PRECISAMENTE :

i) $\{a_n\}$ CRESCENTE $\Rightarrow \lim a_n = \sup a_n$

ii) $\{a_n\}$ DECRESCENTE $\Rightarrow \lim a_n = \inf a_n$

DIM.

$\approx i \approx$

a) $\sup a_n = +\infty \Rightarrow \forall \varepsilon > 0, \exists \nu \in \mathbb{N} : \varepsilon < a_\nu$

$\Rightarrow \forall \varepsilon > 0, \exists \nu \in \mathbb{N} : \forall n > \nu, \quad \varepsilon < a_\nu \le a_n$

$\Rightarrow \lim a_n = +\infty$

b) $\sup a_n = \ell \in \mathbb{R} \iff \begin{cases} 1)\ \forall n \in \mathbb{N},\ a_n \le \ell \\ 2)\ \forall \varepsilon > 0, \exists \nu \in \mathbb{N} : \ell - \varepsilon < a_\nu \end{cases}$

$\Rightarrow \forall \varepsilon > 0, \exists \nu \in \mathbb{N} : \forall n > \nu, \quad \ell - \varepsilon < a_\nu \le a_n \le \ell$

$\Rightarrow$ " " ", $\ell - \varepsilon < a_n < \ell + \varepsilon$

$\Rightarrow \lim a_n = \ell$

□