LIMITE

$$\lim_{n\to+\infty} a_n = \ell \in \mathbb{R} \overset{\text{DEF}}{\iff} \forall \varepsilon>0, \exists \nu \in \mathbb{N}: \forall n>\nu,\ |a_n-\ell|<\varepsilon$$

$$\ell-\varepsilon < a_n < \ell+\varepsilon$$

$$\iff \forall I(\ell)=I(\ell,\varepsilon), \exists \nu\in\mathbb{N}: \forall n>\nu,\ a_n \in I(\ell$$

ESEMPIO 1.- $\lim_{n\to+\infty} \frac{1}{n} = 0$

$$-\varepsilon < \frac{1}{n} < \varepsilon \iff n > \frac{1}{\varepsilon}$$

$\square$

$$\lim_{n\to+\infty} a_n = +\infty \overset{\text{DEF}}{\iff} \forall \varepsilon>0, \exists \nu\in\mathbb{N}: \forall n>\nu,\ \varepsilon < a_n$$

$$\iff \forall I(+\infty), \exists \nu\in\mathbb{N}: \forall n>\nu,\ a_n \in I(+\infty)$$

ESEMPIO 2.- $\lim_{n\to+\infty} n^2 = +\infty$

$$\varepsilon < n^2 \iff \sqrt{\varepsilon} < n$$

$\square$

$$\lim_{n\to+\infty} a_n = -\infty \overset{\text{DEF}}{\iff} \forall \varepsilon>0, \exists \nu\in\mathbb{N}: \forall n>\nu,\ a_n < -\varepsilon$$

$$\iff \forall I(-\infty), \exists \nu\in\mathbb{N}: \forall n>\nu,\ a_n \in I(-\infty)$$

ESEMPIO 3.- $\lim_{n\to+\infty} \log_{1/2} n = -\infty$

$$\log_{1/2} n < -\varepsilon \iff n > \left(\tfrac{1}{2}\right)^{-\varepsilon} \iff n > 2^{\varepsilon}$$

$\square$